THE UNIVERSITY OF ALBERTA.

A DETERMINATION OF GENERALIZATIONS BASIC TO THE MATHEMATICS CURRICULA OF THE INTERMEDIATE AND SENIOR HIGH SCHOOLS OF CANADA.

PART 3: GENERALIZATIONS OF GRADE IX.

A DISSERTATION SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION.

FACULTY OF EDUCATION.

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INTRODUCTION.

Up to the time this analysis was made, no tabulation of the subject matter common to Canadian Mathematics courses was available to educators. To the author of this report the usefulness of this type of tabulation was obvious, and it was agreed that an effort should be made to assist in compiling one for the entire Intermediate and Senior High School Mathematics field of Canada. This report, the third of the series "A Determination of Generalizations Basic to the Mathematics Curricula of the Intermediate and Senior High Schools of Canada", deals with the subject matter of the third year or the Grade Nine Mathematics Curriculum.

The subject matter offered in any mathematics text may be divided into three groups: generalizations, language and information. Although this report deals primarily with generalizations, it was not possible to ignore the other two groups of material. At the beginning of the study an attempt was made to define each of these three terms for the purpose of distinguishing them. However as the study progressed, it became apparent that the simplest solution was that of classifying the concepts arbitrarily either as generalizations, language or information, owing to the difficulty of border-line terms. For example, such terms as: parallel lines, angle, perpendicular were classified as language although each is as rich in ideas as many of the generalizations. The language

terms, arbitrarily selected as such, have been listed alphabetically in Appendix III.

The same difficulty arose with the distinction between generalizations and information. In many cases social concepts or generalizations were difficult to classify as either generalizations or as information, particularly when the author of the text departed from the field of pure mathematics for the purpose of gaining functionality in his presentation. For this reason, a section of this report deals with some of the information or generalizations offered in the more "socialized" texts.

The texts selected for study were those being used in the Grade Nine Mathematics curricula of the nine provinces during 1943-44. With the division of Quebec into Protestant Quebec and Catholic Quebec, ten texts or groups of texts were analyzed. In addition to these texts, the Programmes of Studies for each of the provinces were examined so that only subject matter offered in the classrooms would be tabulated in this report. The texts which were analyzed are listed by provinces in Appendix I. Notes made from the Programmes of Studies are given in Appendix II.

Although the purpose of this report was to tabulate the frequency of the generalizations in these texts, to determine those which were more or less common, it was not possible to overlook completely the varying points of view with which the texts were written and with which it appeared that the mathematics courses were taught in the various

provinces. If some standard were available, it seemed possible to classify the texts in some order from those which placed the most stress on "socialized" mathematics or applications of the generalizations and least stress on formal or "logical" presentation through a number of intermediate steps to those texts which placed most stress on the more rigorous or "logical" presentation of subject matter.

In the tabulation of generalizations an attempt was made to indicate this variation in the method of presentation, that is, whether the generalization in each case was:

(a) assumed to be true, and used without explanation or statement,

(b) assumed to be true, but explained or stated informally,

(c) offered with a proof, or stated formally often with an explanation,

Three numbers are given in this report with each generalization to indicate the number of texts which present it in each of these three ways. For example, the first generalization is presented by no texts as (a), by two texts as (b) and by no texts as (c).

No attempt has been made to arrange the generalizations in a logical or in a psychological order of classroom presentation. The generalizations were arranged in an alphabetical order of main headings merely for the purpose of having some convenient system to use in tabulation[1] This order has been followed in this report. Under each of the

main headings, the generalizations are arranged in the order of frequency, those occurring most frequently being listed first. In some cases the texts are indicated by letter reference so that the reader may note whether the same or different text or text groups offer the successive generalizations. For example, the first generalization is given in texts F and B. No key is given for this letter reference as the author does not wish to draw contrasts between specific provinces.

THE GENERALIZATIONS.

I. Area - (see also Formulas - Area)	(a)	(b)	(c)[1]	
(1) The area of any figure may be found by drawing the figure on squared paper to scale and counting the number of square units contained by the figure.	0	2	0	FB
(2) Parallelograms on the same base and between the same parallels (or of the same altitude) are equal in area.	0	0	1	F
(3) The area of a parallelogram is the same as that of a rectangle on the same base and between the same parallels.	0	0	1	F
(4) Parallelograms on equal bases and of the same altitude are equal in area.	0	0	1	F
(5) Triangles on the same base and between the same parallels are equal in area.	0	0	1	F
(6) If a triangle and a parallelogram stand on the same base and between the same parallels, the area of the triangle is half that of the parallelogram.	0	0	1	F

1. (a) assumed to be true, but used without explanation or statement,
   (b) assumed to be true, but explained or stated informally,
   (c) offered with a proof, or stated formally often with an explanation.

II. <u>Angles</u> - (see also Triangle-angles) (a)-(b)-(c)

(1). If two straight lines intersect, the vertically opposite angles are equal. 0 - 1 - 5

(2). When one straight line meets another straight line, the sum of the adjacent angles is equal to two right angles. 0 - 0 - 5

(3). If two adjacent angles are together equal to two right angles, the exterior arms of the angles form one straight line. 0 - 2 - 3

(4). If any number of straight lines meet at a point, the sum of the adjacent angles is equal to four right angles. 0 - 2 - 3

(5). Complements of the same or of equal angles are equal. 0 - 0 - 3

(6). Supplements of the same or of equal angles are equal. 0 - 0 - 3

(7). The internal and external bisectors of an angle are at right angles to each other. 0 - 2 - 0

III. <u>Assumptions</u> - Geometric (see also Equations-axioms)

(1) Things which are equal to the same thing are equal to one another. 0 - 0 - 4 - FCGK

(2). Playfair's axiom - 0 - 0 - 3

(a) Two intersecting straight lines cannot beth be parallel to the same straight line. -CGK

(b) Through a given point there can be only one straight line parallel to a given straight line. - GK

(3) Magnitudes which can be made t8 coincide are equal to each other. 0 - 0 - 3 - CGK

(4) Propositions may be taken as assumptions (as facts accepted without formal proØf) or as theorems (for which formal proof is given). 0 - 2 - 0 - FC

(a)-(b)-(c)

(5).	The whole is greater than its part and equal to the sum of all its parts.	0 - 0 - 2 - FC
(6)	If equals are added to unequals, the sums are unequal.	0 - 0 - 2 - FC
(7)	If equals are subtracted from unequals the remainders are unequal.	0 - 0 - 2 - FC
(8)	There can be only one straight line joining two given points, or two straight lines cannot enclose a space.	0 - 0 - 2 - GK
(9)	Only one perpendicular can be drawn to a given straight line from a point on the line.	0 - 0 - 2 - GK
(10)	All right angles are equal.	0 - 0 - 2 - GK
(11)	Every finite straight line has a point of bisection.	0 - 0 - 2 - GK
(12)	Every angle may be supposed to have a line of bisection.	0 - 0 - 2 - GK
(13)	A line may be drawn through a given point parallel to a given line.	0 - 0 - 2 - GK
(14)	If the first of three quantities is greater than the second, and the second is greater than the third, then the first is greater than the third.	0 - 0 - 1 - F
(9),(11),(12),(13)	above are also given as hypothetical constructions.	0 - 0 - 2 - GK

IV. <u>Averages</u>

The sum of the deviations of a set of numbers from their average is zero. 0 - 0 - 1

V. <u>Converses</u>

A converse of a theorem may or may not be true. 0 - 2 - 0

VI. Constructions. (a)-(b)-(c)

(1) A straight line segment may be bisected by joining the common points of two equal intersecting circles whose centres ate at the ends of the given line segment. 0 - 4 - 4 -

Two[1] of the texts listed under (c) also give this generalization without formal proof, but have not been included among the four listed under (b). - FC

(2) A perpendicular may be drawn to a given line at a point in the line by constructing with arcs congruent triangles of which the perpendicular is the common side. 0 - 5 - 3 -

One of the texts listed under (c) also gives this construction informally. - C

(3) A perpendicular may be drawn to a given line from a given point outside the line. 0 - 5 - 3 -

Here as in (2) the common method is that of using arcs to get congruent triangles of which the perpendicular is the common side. As in (2) one of the texts gives this construction both formally and informally, and the same text uses a set square to make the construction. Another text uses three methods, variations of the arcs or congruent triangle method and offers a proof for each. - C - G

(4) An angle may be drawn equal to a given angle,(constructing congruent triangles). 0 - 5 - 3

One of the texts gives this construction both formally and informally. - C

(5) Through a given point one straight line may be drawn parallel to a given straight line. 0 - 5 - 3

The methods used for this construction vary, such as: by alternate angles, making a parallelogram, by corresponding angles, using two set squares. One of the texts gives this construction both formally and informally. - C

1. In no case is a text listed under two classifications. If the text is listed under (c) it is not given under (b). This has been done so that the reader may know the number of texts or text groups which deal with the generalization.

	(a)-(b)-(c)
(6) Any angle may be bisected by making equal segments of its arms the corresponding sides of two congruent triangles which have their common side between the arms of the given angle.	0 - 4 - 3

One text gives this construction both formally - C and informally.

(7) A given straight line segment may be divided into a number of equal parts.	0 - 3 - 3

For this construction nearly all the texts use the method of parallels.

(8) A triangle may be constructed if given:

(a) three sides	1 - 4 - 2
(b) two sides and contained angle	1 - 5 - 0
(c) two angles and one side	1 - 5 - 0
(d) two sides and the angle opposite one side.	0 - 1 - 2
(9) A square may be described on a given line segment.	1 - 0 - 3
(10) A perpendicular bisector may be drawn to a given line segment.	0 - 3 - 1

The text which gives this construction formally - C also gives it informally.

(11) A right-angled triangle may be drawn if given the hypotenuse and one side.	0 - 1 - 2
(12) A parallelogram may be drawn if given two adjacent sides and the included angle.	0 - 1 - 2

(13) A convex quadrilateral may be drawn if given:

(a) the length of four sides and one angle,	0 - 1 - 2
(b) the length of three sides and two contained angles,	0 - 1 - 0 - H
(c) two adjacent sides and any three angles.	0 - 1 - 0 - H
(14) The centre of a given circle may be found by locating the point of intersection of the right bisectors of two chords.	0 - 2 - 0 - HD

(a)-(b)-(c)

(15) A triangle may be drawn equal in area to a given quadrilateral. 0 - 0 - 1 - F

(16) A line segment may be divided in the ratio of two given lines. 0 - 1 - 0 - H

(17) A circle may be circumscribed about a given triangle. 0 - 1 - 0 - H

(18) A circle may be inscribed in a given triangle. 0 - 1 - 0 - H

(19) A tangent may be drawn to a given circle. 0 - 1 - 0 - D

(20) A square may be circumscribed about a given circle. 0 - 1 - 0 - D

VII Circles

(1) The areas of concentric circles vary directly as the squares of the radii. 1 - 0 - 0 - F

(2) If two angles at the centre of a circle are equal, they cut off equal arcs on the circle. 0 - 1 - 0 - B

(3) The area of a sector of a circle is to the area of the circle as the:

(a) angle of the sector is to 360° 0 - 0 - 1 - F

(b) the arc of the sector to the circumference. 0 - 0 - 3 - FGE

(4) The centres of all circles passing through two fixed points lie on the perpendicular bisector of the straight line segment joining the points. 1 - 0 - 1 - DH

This is similar to construction (14) page 8.

(5) Through three fixed points, not in a straight line, one and only one circle may be drawn. 0 - 0 - 1 - H

VIII Casting out nines - arithmetic checking - (check figure)

The remainder found when any number is divided by nine is the same as the remainder found when the sum of the digits of the number is divided by nine. 1 - 0 - 1 - DA

IX Decimals (Arithmetic) (a)-(b)-(c)

(1) Decimals may be multiplied by multiplying the given numbers as if no decimal point were present, placing the decimal point in the product so that the number of digits to the right of it is the sum of the number of digits to the right of the decimal point in the multiplier and in the multiplicand. 0 - 2 - 2 - HBDA

(2) Decimals may be divided by decimals by moving the decimal point the same number of places to the right in the divisor and the dividend, making the divisor a whole number, dividing in the usual way, placing the decimal point in the quotient immediately after bringing down the first digit to the right of the decimal point in the dividend. 0 - 3 - 1 - HBDA

(3) A decimal may be changed to its equivalent vulgar fraction by using for the numerator the number with the decimal point ommitted and for the denominator the symbol "1" followed by as many zeros as there are digits on the right of the decimal point. 1 - 0 - 1 - DA

(4) A fraction may be expressed either as a terminating or recurring decimal by dividing the numerator of the fraction by the denominator. 1 - 0 -1 - DA

(5) A recurring decimal expressed as a fraction, or a mixed recurring decimal expressed as a mixed number. 0 - 1 - 0 - D

X Divisibility Tests (Arithmetic)

A number is divisible by

(a) two - if it is an even number, 0 - 0 - 1 - A
(b) five- when the number ends in five or zero, 0 - 0 - 1 - A
(c) four- when the number ends in two digits divisible by four 0 - 0 - 1 - A
(d) three or nine- when the sum of the digits is divisible by three or nine. 0 - 0 - 1 - A

XI Evaluation A number may be put in the place of a letter in a given algebraic expression to find the corresponding numerical value of the expression. 1 - 6 - 1

XII <u>Equations</u> (a)-(b)-(c)

(1) Axioms:

(a) Addition - If equals are added to equals, the sums are equal. 0 - 0 - 10

(b) Subtraction - If equals are subtracted from equals, the remainders are equal. 0 - 0 - 10

(c) Multiplication - If equals are multiplied by equals, the products are equal. 0 - 0 - 10

(d) Division - If equals are divided by equals, the quotients are equal. 0 - 0 - 10

(2) "We must do the same thing to each side of an equation." 6 - 1 - 2

(3) In solving an equation the unknown number is placed on one side and the known numbers on the other side. 7 - 1 - 1

(4) Transposition or transformation - A term may be dropped from either member of an equation, provided that the corresponding opposite term is written in the other member; or a term may be removed from one side to the other by changing its sign. 0 - 0 - 8

(5) When solving an equation, do the addition or subtraction if any, then the multiplication if any, then the division if any. 6 - 0 - 1

(6) Equations may be solved by graphs as:

(a) motion problems, 0 - 1 - 0 - F

(b) two simultaneous equations (not inconsistent nor dependent) may be solved by finding the co-ordinates of the common point of their graphs. 1 - 1 - 1

(7) If the same term preceded by the same sign occurs in both members of an equation, it may be cancelled or dropped from both members. 0 - 0 - 2

(8) The signs of all terms of an equation may be changed without destroying the equality. 0 - 0 - 2

 (a)-(b)-(c)

(9) Two equations in two unknowns can have a definite solution only when the equations are independent and consistent. 0 - 1 - 1

(10) Simultaneous linear equations in two unknowns may be solved:

(a) by elimination of one of the unknowns. If the coefficients of one unknown have the same absolute value, by addition or subtraction. If not, by multiplying the equations by such numbers as will make the coefficients of one unknown have the same absolute value, then eliminating that unknown by addition or subtraction. 0 - 4 - 1

(b) by reducing to standard form and eliminating one unknown. 0 - 2 - 0

(c) by substituting the value of one unknown in terms of the other. 0 - 3 - 0

(d) by comparing the values of one unknown. 0 - 1 - 0

(11) Simultaneous equations in three unknowns may be solved by elimination. 0 - 1 - 0

(12) Some quadratic equations may be solved by factoring - by reducing the equation to standard form, factoring, equating each factor to zero, and solving the simple equations resulting. 0 - 3 - 1

In some problems, some of the solutions obtained in this manner are not applicable. 0 - 1 - 0

(13) An equation of the second degree contains one term at least with the second degree of the literal factor. 0 - 1 - 2

(14) Equations of the first degree have only one root; of the second degree only two roots. 1 - 0 - 0

(15) It is necessary to attach the double sign to the square root of only one side of an equation when the square root is found. 0 - 0 - 4

XIII <u>Functionality</u> (a)-(b)-(c)

(1) Relationships between variables may be expressed by a table, by a formula, by a graph. 0 - 3 - 2

(2) A formula may be made from a graph by making a table of values from the graph and finding the formula which fits the table of related values. 0 - 0 - 1

(3) A formula may be made from a table of values. 0 - 1 - 0

XIV <u>Rules of Fundamental Operations.</u>

(1) Rules of addition and subtraction:

(a) The sum of unsigned similar terms may be found by finding the sum of the coefficients of the given terms and multiplying it by the literal factor appearing in each of the terms. 0 - 3 - 5

(b) The sum of signed similar terms may be found in the same manner: 0 - 0 - 3

(i) To add several signed numbers having like signs, find the sum of their absolute values and prefix to this sum their common sign. 0 - 0 - 9

(ii) To add two signed numbers having unlike signs, find the difference between their absolute values and prefix to it the sign of the number which has the greater absolute value. 0 - 0 - 9

(iii) To add a negative number is the same as subtracting a positive number of the same absolute value. 0 - 0 - 5

(iv) To add several signed numbers find the sum of all the positive numbers, and the sum of all the negative numbers and finally the sum of the results. 0- 5 - 2

(c) Dissimilar terms cannot be added. 1 - 8 - 0

	(a)-(b)-(c)
(d) Rule of order - In addition the addends may be combined in any order.	0 - 0 - 4
(e) Rule of grouping - In addition the addends may be grouped in any order.	0 - 0 - 1
(f) The difference of unsigned similar terms may be obtained by finding the difference between the coefficients of the given terms and multiplying it by the literal factor appearing in each of the terms.	0 - 4 - 3
(g) The difference of signed similar may be found in the same manner:	0 - 2 - 1
(i) One signed number may be subtracted from another by changing mentally the sign of the subtrahend and proceeding as in addition.	0 - 0 - 9
(h) Dissimilar terms cannot be subtracted.	1 - 7 - 0
(i) Polynomials may be added by adding similar terms:	
(i) adding polynomials with like signs.	0 - 9 - 0
(ii) adding polynomials with unlike signs.	0 - 9 - 0
(j) Polynomials may be subtracted by subtracting similar terms:	
(i) subtracting polynomials with like signs.	0 - 6 - 0
(ii) subtracting polynomials with unlike signs.	0 - 9 - 0
(k) Casting out nines (check figure method) may be used in arithmetic to check addition and subtraction.	0 - 1 - 0

(2) Rules of multiplication: (a)-(b)-(c)

(a) A polynomial may be multiplied by a monomial by multiplying each term of the polynomial by the monomial, uniting the resulting terms by their proper signs. 0 - 6 - 4

(b) Two signed numbers may be multiplied by finding the product of their absolute values and

(i) making the product positive if the signed numbers have like signs. 0 - 1 - 8

(ii) making the product negative if the signed numbers have unlike signs. 0 - 1 - 8

(c) If the number of negative factors in the product of several signed numbers is even, the product is positive, if odd, the product is negative. 0 - 2 - 4

(d) Rules of exponents-

(i) a^2 means a times a 0 - 8 - 1

(ii) The exponent of the product of powers of like bases is the sum of the original exponents. 0 - 1 - 8

(iii) An exponent of one is not written, but is understood. 2 - 2 - 1

(iv) The zero power of any number equals one. 0 - 0 - 1

(e) The product of two or more monomials may be found by multiplying the numerical coefficients and this product by the product of the literal factors. 0 - 1 - 8

(f) Sequence of factors - When there are two factors, one numerical and one literal, the numerical factor is written first. 4 - 3 - 1

(g) When there is no sign shown between literal numbers, or between a numerical and literal factors, multiplication is understood. 4- 1 - 3

(h) When no numerical coefficient is given, a coefficient of one is understood. 1 - 3 - 3

(a)-(b)-(c)

(i) A number and a monomial may be multiplied by multiplying the numerical coefficient by the number and the product by the literal coefficient. 5 - 2 - 0

(j) A polynomial may be multiplied by a polynomial by multiplying each term of one by each term of the other and adding the partial products. 0 - 1 - 7

(i) by using detached coefficients 0 - 2 - 0

(k) Rule of order - In multiplication the factors may be arranged in any order. 0 - 0 - 8

(l) Rule of grouping - In multiplication the factors may be grouped in any order. 0- 1- 1

(m) In arithmetic, multiplication may be checked by casting out nines (check figure method). 0 - 2 - 0

(3) Rules of multiplication - Special products usually associated with factoring

(a) The square of a monomial is found by squaring the numerical coefficient and affixing to it the product of the literal coefficients with each exponent doubled. 4 - 3 - 2

(b) In the product of a binomial by a binomial of the form ax + b, the first term of the product is the product of the first terms of the binomials, the middle term of the product is the algebraic sum of the cross products, the third term of the product is the product of the second terms of the binomials. 0 - 4 - 5

(c) The product of the sum and the difference of the same two numbers equals the difference of their squares. 0 - 0 - 9

(d) The square of a binomial consists of the square of the first term plus twice the product of the two terms plus the square of the second term. 1 - 0 - 8

(a)-(b)-(c)

(e) If the product of two or more factors is zero, then one or more factors equals zero. 2 - 0 - 3

(f) The square of a polynomial is equal to the sum of the squares of each of its terms together with twice the sum of the products of each pair of terms. 0 - 0 - 2

(4) Rules of Division-

(a) A monomial may be divided by a number by dividing the numerical coefficient by the number and multiplying this quotient by the literal factor. 2 - 7 - 0

(b) One signed number may be divided by another by finding the quotient of their absolute values and 0 - 4 - 5
 (i) making the quotient positive if the signed numbers have like signs,
 (ii) or making the quotient negative if the signed numbers have unlike signs.

(c) Rule of exponents - The exponent of the quotient of powers having like bases is the difference of the original exponents. 0 - 1 - 8

(d) A monomial may be divided by a monomial by dividing one numerical coefficient by the other, dividing the literal coefficients, expressing the result as the product of these two quotients. 1 - 5 - 4

(e) A polynomial may be divided by a monomial by dividing each term of the polynomial by the monomial and uniting the resulting terms by their proper signs. 0 - 2 - 7

(f) The product of two or more factors may be divided by a number by dividing any one and only one factor by that number. 1- 5 - 1

(g) Division by zero is impossible. 0 - 0 - 3

(h) When a number is divided by itself the quotient is one. 0 - 0 - 2

(a)-(b)-(c)

(i) A polynomial may be divided by a polynomial by arranging the terms of both the dividend and the divisor according to the descending or ascending powers of the same letter, dividing the first term of the dividend by the first term of the divisor obtaining the first term of the quotient, multiplying the divisor by the first term of the quotient and subtracting this product from the dividend, thus obtaining a new dividend, and repeating these steps as long as division is possible. 0 - 5 - 1

(i) using detached coefficients 0 - 2 - 0 - GK

(ii) with inexact division 0 - 3 - 0 - GJK

(j) In arithmetic, division may be checked by casting out nines(check figure method). 0 - 2 - 0

XV <u>Formulas</u> -

(1) In any formula, the letters stand for numbers. 2 - 2 - 1

(2) Linear formulas-

(a) The perimeter of a square equals four times one side $p = 4s$. 1 - 0 - 3

(b) The circumference of a circle $C = \pi d = 2\pi r$. 1 - 1 - 5

(c) The perimeter of a rectangle $p = 2(a + b)$. 1 - 2 - 1

(d) In any circle $d = 2r$. 1 - 2 - 1

(e) The length of the perpendicular to the longest side of a triangle from the opposite vertex (given from an illustration). 0 - 2 - 0

(3) Area formulas-

(a) The area of a rectangle is the product of the length and the width $A = lw$. 2 - 0 - 7

(b) The area of a parallelogram is the product of the base and the altitude $A = bh$. 1 - 0 - 6

(a)-(b)-(c)

(c) The area of a trapezium of parallel sides, b and c and altitude h is $A = \frac{1}{2}h(b + c)$ — 0 - 1 - 6

(d) The area of the curved surface of a cylinder S of circumference c and altitude h is $S = ch = \pi dh = 2\pi rh$ — 0 - 1 - 6

(e) The area of a triangle A of base b and altitude h is $A = \frac{1}{2}bh$. — 1 - 0 - 5

(f) The area of a circle A of radius r is $A = \pi r^2$ — 1 - 0 - 5

(g) The area of a square A of side s is $A = s^2$. — 1 - 1 - 3

(h) The total surface area of a cube S of side e is $S = 6e^2$. — 1 - 1 - 0 - HB

(i) The area of the whole surface of a cylinder A, radius r, height h is $A = 2\pi rh + 2\pi r^2$. — 0 - 2 - 1 - EGA

(j) The area A of a flat ring, inner radius r and outer radius R is $A = \pi(R + r)(R - r)$ — 0 - 0 - 1 - D

(k) The area S of the surface of a sphere of radius r is $S = 4\pi r^2$. — 0 - 1 - 2 - BDG

(l) The area of the curved surface of a cone of slant height S and radius r is $A = \pi rs$. — 0 - 1 - 1 - BD

(m) The area L of the lateral surface of a regular pyramid of perimeter p and slant height s is $L = ps/2$. — 0 - 0 - 1 - D

(n) The area of a quadrilateral is one-half the product of a diagonal and the sum of the perpendicular lengths to that diagonal from the other vertices. — 0 - 0 - 1

(o) The area of a triangle of sides a,b,c, where s is half the perimeter is $\sqrt{s(s-a)(s-b)(s-c)}$ — 0 - 0 - 3 - EGK

(4) Volume formulas-	(a)-(b)-(c)
(a) The volume V of a rectangular solid of sides l,w,h is $V = lwh$.	1 - 0 - 6
(b) The volume V of a cube of side e is $V = e^3$.	1 - 1 - 4
(c) The volume V of a cylinder of height h and radius r is $V = \pi r^2 h$.	0 - 1 - 5
(d) The volume V of a pyramid of base b and height h is $V = \frac{1}{3} bh$	0 - 1 - 3
(e) The volume V of a sphere of radius r is $V = \frac{4}{3}\pi r^3$	0 - 1 - 3
(f) The volume V of a prism of base area b and altitude h is $V = bh$.	0 - 1 - 2
(g) The volume V of a cone of base radius r and altitude h is $V = \frac{1}{3}\pi r^2 h$.	0 - 1 - 2
(h) The volume V of a cylinder wall of inner radius r and outer radius R, altitude h is $V = \pi(R+r)(R-r)$	0 - 1 - 1

(5) Other formulas-	
(a) Simple interest $i = prt$. interest i, principal p, rate r, time t.	0 - 0 - 6
(b) Compound interest $A = p(1+r)^n$ amount A, principal p, rate r, number of periods n.	0 - 0 - 1
(c) Temperature $F = 32 + \frac{9}{5}C$	0 - 0 - 3
and $C = \frac{5}{9}(F - 32)$	0 - 0 - 1
(d) Distance $d = rt$, distance d, rate r, time t.	0 - 0 - 2
(e) Percent $P = RB$, rate R, base B	0 - 0 - 1
(f) Selling price = Cost + Margin Profit = Margin - Overhead.	0 - 0 - 2

The above are examples only of the formulas which are given in the texts for illustration purposes.

XVI <u>Factoring</u> (a)-(b)-(c)

(1) A monomial may be factored as the product of its prime factors. 0 - 6 - 1

(2) A monomial factor may be removed from a polynomial by dividing each term of the polynomial by the largest common monomial factor, the monomial being one factor, and the quotients the other factor. 0 - 5 - 3

A binomial factor may be removed from a polynomial by grouping. 0 - 2 - 0

(3) The difference of two squares may be expressed as the product of two factors, one the sum of the square roots, the other the difference of the square roots of the numbers which are squared. 0 - 0 - 8

(4) The square root of a square monomial may be found by multiplying the square root of the numerical coefficient by the square root of the literal coefficients, (the same literal factors to one-half their original exponents). 0 - 4 - 3

(5) A trinomial may be expressed as the product of two binomials:

(a) by a general method of inspection by cross multiplication where the coefficient of x^2 is one or greater than one. 0- 7 - 0

(b) by decomposition, by dividing the middle term into two terms and factoring by grouping. 0 - 2 - 0

(c) by recognition of three types:

(i) where all the signs of the trinomial are positive, the sign of the second term of each binomial is plus. 0 - 3 - 1 - DACJ

(ii) where is the sign of the middle term of the trinomial is the only negative sign of the trinomial, the sign of the second term of each binomial is minus. 0 -3 -1 - DACJ

(a)-(b)-(c)

(iii) where the third term of the trinomial is negative, the signs of the second terms of the binomial are unlike, the sign of the larger cross product is the same as the sign of the middle term of the trinomial. 0 - 2 - 2 - DACJ

(6) A trinomial square may be expressed as the square of a binomial in which the terms are the square roots of the first and last terms of the trinomial, with the sign of the binomial that of the middle term of the trinomial. 0 - 6 - 1

(7) The sum of two cubes may be expressed as the product of the sum of the two numbers cubed and the square of the first minus their product plus the square of the second. 0 - 0 - 2 - GK
The difference of two cubes may be expressed as the product of the difference of the two numbers cubed and the square of the first plus their product plus the square of the second.

(8) The square of any trinomial or polynomial may be factored. 0 - 2 - 0 - GK

(9) Incomplete squares may be factored 0 - 2 - 0 - GK

(10) When factoring completely, the largest monomial common factor is removed first and the resulting expression or quotient factored. 0 - 1 - 1 - JC

(11) Factor theorem - Any expression is divisible by (x-a) if it equals zero when a is substituted for x. 0 - 0 - 2 - GK

When (x-1) is a factor, the sum of the coefficients equals zero. 0 - 2 - 0 - GK

XVII <u>Fractions</u> [1]

(1) Unlikefractions may be added or subtracted by changing them to like fractions first. 1ar- 3 - 1
1al 3ar

1. In this section, the frequencies are broken down into those texts which apply the generalization to both Algebra and Arithmetic (no symbol),to Arithmetic only (ar), to Algebra only (al). The total of the frequencies is the total of the texts which present the generalization.

(2) The numerator and the denominator of a fraction may be divided or multiplied by the same number, except zero, without changing the value of the fraction. 0 - 1al-6 2ar

(i) $\frac{a}{b} = \frac{ma}{mb}$; $\frac{na}{nb} = \frac{a}{b}$ 0 - 1al- 1 1al

(ii) When the sign of both terms, or the sign of one term and the sign of the whole fraction is changed, the value of the fraction is not changed. 0 - 0 - 1al

(3) Fractions may be expressed in their lowest terms by dividing both terms by all the common factors. 1ar- 1 - 4 1ar

(i) Improper fractions may be expressed as whole or mixed numbers 1ar- 1 - 1ar 1ar

(ii) In Algebra, fractions may be expressed in their lowest terms by factoring completely both terms, dividing by all the common factors, and multiplying out the quotients of the numerators to form the new numerator, and the quotients of the denominator to form the new denominator. 0 - 1al - 4al

(iii) In Algebra, fractions may be divided into the classes: 0 - 1al - 0
- those with unlike monomial denominators,
- those with unlike binomial denominators,
- those with unlike binomial denominators and numerators.

(4) The product of two or more fractions may be expressed as the product of their numerators divided by the product of their denominators. In Algebra factor first the numerators and denominators. 1ar-3 - 2 3ar

(5) A number or fraction may be divided by a fraction by inverting the divisor and multiplying. In Algebra factor first the numerator and the denominator. 1ar-3 - 2 3ar

(6) A mixed number may be expressed as an improper fraction by multiplying the whole number by the denominator, adding the numerator, and putting this sum over the denominator. 0 - 2 - 1ar 1ar 3al

(a)-(b)-(c)

(7) Like fractions may be added or subtracted by adding or subtracting the numerators and putting the result over the common denominator. lar- lar- 3 - lar

(i) special case of denominators differing in sign only, (b-a) = -(a-b) 0 - 1 - 0

(8) A complex fraction may be simplified by simplifying the numerator and the denominator separately, and then dividing. 0 - lar - lal - lal

XVIII <u>Highest Common Factor</u>

The highest common factor (HCF) of any two numbers is a factor of their difference. 0 - 0 - 1

XIX <u>Graphical Representation</u>

(1) Variables may be represented in graphs by:

(a) Broken line graphs - in which the heights of the points so connected represent the values of the variable. 0 - 8 - 0

(i) Straight line graphs of the above. -1 - 0

(b) Bar graphs - in which the lengths of the bar represent the values of the variable. 1 - 4 - 1

(c) Circle graphs - in which the areas of the sectors or the size of the central angles represent the values of the variable. 0 - 5 - 1

(d) Curved line graphs - in which the ordinates of the points so connected represent the values of the variables. 0 - 4 - 0

(i) Interpolation may be done in these graphs. 1 - 1 - 0

(e) Pictographs - in which the heights or sizes of the pictures represent the values of the variable. 1- 3- 0

(a)-(b)-(c)

(f) Rectangular Distribution Graphs may be used to show how a bar graph may be composed of several sub-variables. 0 - 2 - 0

(2) Co-ordinate graphs-

(a) In a co-ordinate graph system, the abscissa is read first. 1 - 0 - 2 - DBJ

(b) (i) Any point on the graph of an equation has co-ordinates that satisfy the equation. 0 - 1 - 1 - JD

(ii) Any point not on the graph of the equation has co-ordinates that do not satisfy the equation. 0 - 0 - 1 - D

(iii) Any pair of values of x and y that satisfy the equation represent a point on the graph of the equation. 0 - 0 - 1 - D

(iv) Any pair of values that do not satisfy the equation represent a point not on the graph. 0 - 0 - 1 - D

(c) The graph of a linear or a first degree equation having two variables is a straight line. 0 - 1 - 1 - DJ

(d) The graph of a linear equation is located by two points on it. 0 - 1 - 1 - DJ

XX Loci

(1) The locus of all points equally distant from two given points is the right bisector of the straight line joining the points. 0 - 1 - 1 - FG

(2) Any point on the bisector of an angle is equidistant from the arms of the angle. 0 - 0 - 2 - FG

(3) The locus of a point at a fixed distance from one given point is a circle. 0 - 1 - 0 - F

(4) Loci in space -

(i) The locus of a point equally distant from a given point is a sphere. 0 - 1 - 0 - F

(ii) The locus of a point equally distant from a given line is a cylinder. 0 - 1 - 0 - F

(a)-(b)-(c)

(iii) The locus of a point equally distant from two given points is a plane. 0 - 1 - 0 - F

XXI <u>Longitude and Time</u>

(1) For every 15° of longitude that places are apart, there is a difference in time of one hour. 0- 0 - 3 - HDJ

(2) Clocks are faster to the east and slower to the west. The difference in time between any two adjacent time belts is one hour. 0 - 2 - 1 -HDJ

XXII <u>Measurement</u>

(1) Quantities may be measured by standard units of measurement. 1 - 2 - 0 - DBA

(2) The possible error may be expressed for a measurement. 0 - 2 - 1 - DBA

(3) Distances and dimensions may be found by indirect measurement:

(a) by scale drawing. 0 - 1 - 0 - D

(b) by using the Pythagorean theorem to find unknown sides. 0 - 1 - 1 - DB

(c) by using the facts of similar triangles. 0 - 1 - 0 - D

(d) by using trigonometric functions such as the tangent, the sine, the cosine. 0 - 1 - 0 - D

(4) The number of board feet in lumber may be found by multiplying the length in feet by the width in inches by the thickness in inches and dividing the result by 12. 0 - 0 - 1 - H

(a) or the area in square feet by the thickness in inches. 0 - 0 - 1 - E

(b) In a tapering board, the average width is used. 0 - 0 - 1 - E

(5) Land may be measured by chains. 0 - 1 - 0 - H

(6) 1000 shingles laid 4" to the weather cover one square (100 sq.ft.). 880 shingles laid 4½" to the weather cover one square. 0 - 0 - 1 - E

(a)-(b)-(c)

(7) In lathing and plastering only one-half the area of openings is subtracted from the total area. 0 - 0 - 1 - E

(8) Metric system-

(a) Advantages-

(i) Easy to use, 0 - 0 - 1 - H

(ii) Eliminates reduction, addition subtraction, multiplication and division of compound denominate numbers, 0 -0 - 2 - HG

(iii) All arithmetic operations are the same as for simple numbers. 0- 0 - 1 - G

(iv) Its common use would facilitate commerce. 0- 0- 2 - HG

(b) Disadvantages-

(i) Conversion to the English system is difficult. 0 - 0 - 2 - HG

(c) Some equivalents

1 centimetre equals 2/5 inches	0 -1 - 5
1 metre equals 39.37 inches	0-0 - 5
1 kilometre equals 5/8 miles	0 - 0 - 5
1 litre equals 8/9 quarts	0 - 0 - 4
1 gallon equals 4½ litres	0 - 0 - 2
1 kilogram equals 2.2 lbs.	0 - 0 - 5
1 metric ton equals 2200 lbs.	0 - 0 - 4
1 c.c. of water weighs one gram	0- 0 - 4
1 litre equals 1000 gm or 1 kg.	0 - 0 - 3
1 hectoare equals 2½ acres	0 - 0 - 3
3 7/10 steres equals 1 cord	0 - 0 - 2

(d) Other equivalents

1 American gallon equals 5/6 Imperial gallons	1 - 0 - 2
1 English ton equals 2240 lbs.	0 - 0 - 1

XXIII <u>Nomenclature</u>.

(1) Algebra uses the letters of the alphabet for the purposes of mathematical shorthand. 3 - 7 - 0

(2) Arithmetic and algebra use signs to abbreviate work. 5 - 5 - 0

XXIV <u>Order of Terms</u> (a)-(b)-(c)

(1) The terms of an expression may be arranged in

(a) ascending powers of the same letter 0 - 1 - 1 - FH

(b) descending powers of the same letter 0- 2 - 1 - FBH

XXV <u>Order of Operations</u>

(1) Operations are performed in the following order: 0 - 0 - 1 - F

(a) find the numerical value of the indicated powers,

(b) perform the indicated multiplications and divisions in the order in which they occur,

(c) perform the indicated additions and subtractions.

or (a) simplify the parentheses, then perform (b),(c) above. 0 - 0 - 2 - FJ

XXVI <u>Percentage</u>

(1) A percent may be changed to a decimal fraction by moving the decimal point two places to the left and dropping the percent sign. 1 - 1 - 2 - DABJ

(2) A decimal fraction may be changed to a per cent by moving the decimal point two places to the right and putting in the per cent sign. 1 - 1 - 2 - DABJ

(3) There are three types of per cent problems:

(a) finding the per cent of a number by changing the per cent to a decimal or common fraction and multiplying it by the number. 0 - 3 - 2 - HDABJ

(b) finding what per cent one number is of another number, by finding the fractional part one number is of the other, and changing this to a decimal fraction and then to a percent. 1 - 3 - 1 - BHDAJ

(c) finding a number when a percent of it is given. 1 - 4 - 0 - BHDAJ

XXVII <u>Pythagorean Theorem</u> (a)-(b)-(c)

In any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. 0 - 1 - 7

XXVIII <u>Parentheses - Brackets</u>

(1) $4(w+2)$ has the same value as $4w+8$ 7 - 1 - 1

(2) When expanding, each term within the parentheses is multiplied by the factor of the parentheses:

(a) When parentheses are preceded by a plus sign, signs of the terms within the parentheses are not changed. 3 - 2 - 4

(b) When parentheses are preceded by a minus sign, the sign of each term within the parentheses is changed. 0 - 2 - 7

(c) If one set of parentheses occurs within another, remove them one at a time, starting with the innermost. 0 - 3 - 2

(3) Numbers may be grouped by using parentheses:

(a) to indicate the order of operations 2 - 6 - 0

(b) to make binomials from polynomials and such. 0 - 5 - 0

(4) In division the line between the numerator and the denominator takes the place of parentheses. 4 - 0 - 4

(5) Any parentheses without an indicated coefficient may be considered to have a coefficient of one. 3 - 0 - 2

XXIX <u>Postulates</u>

(1) A straight line may be drawn from any one point to any other point. 0 - 0 - 3 - CGK

(2) A terminated straight line may be produced any distance in that straight line. 0 - 0 - 3 - CGK

(a)-(b)-(c)

(3) A circle may be described with any point as centre and with any radius. 0 - 0 - 3 - CGK

XXX. Parallel Straight Lines

(1) When a straight line cuts two parallel straight lines:

(a) the alternate angles are equal, 1 - 2 - 5

(b) the interior angles on the same side of the cutting line are together equal to two right angles, 2 - 1 - 5

(c) the corresponding angles are equal. 2 - 1 - 5

(2) When a straight line cuts two other straight lines, the two straight lines are parallel if:

(a) a pair of alternate angles are equal, 0 - 0 - 5

(b) a pair of interior angles on the same side of the cutting line are together equal to two right angles, 0 - 0 - 5

(c) a pair of corresponding angles are equal. 0 - 0 - 5

(3) Straight lines which are perpendicular to the same straight line are parallel to each other. 0 - 0 - 4

(4) The straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side. 0 - 2 - 1

(5) The straight line which joins the middle points of two sides of a triangle is parallel to the third side and is equal to one-half the third side. 0 - 3 - 0

(6) If three or more parallels intercept equal segments on one transversal, they intercept equal segments on every other transversal. 0 - 0 - 2

(i) The above applied to a triangle 0 - 0 - 1

(7) Lines parallel to the same straight line are parallel to each other. 0 - 0 - 2

XXXI <u>Parallelograms</u> (a)-(b)-(c)

(1) In a parallelogram:

(a) the opposite sides are equal, 1 - 2 - 4

(b) the opposite angles are equal, 0 - 1 - 2

(c) each diagonal bisects the parallelogram, 0 - 2 - 4

(d) the diagonals bisect each other. 1 - 1 - 4

(2) A quadrilateral is a parallelogram if

(a) both pairs of opposite angles are equal, 0 - 2 - 1 - GKF

(b) both pairs of opposite sides are equal, 0 - 2 - 1 - GKF

(c) the diagonals bisect each other, 0 - 3 - 1 - GKCF

(d) one pair of opposite sides are equal and parallel. 0 - 0 - 1 - F

(3) If one angle of a parallelogram is a right angle, all the angles are right angles. 0 - 2 - 2 - GKAF

(4) The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are themselves equal and parallel. 0 - 0 - 2 - GK

(5) The sum of the interior angles of a parallelogram equals four right angles. 0 - 0 - 1 - D

(6) If two adjacent sides of a parallelogram are equal, all its sides are equal. 0- 0-1 - F

(7) Squares and rectangles -

(a) The diagonals of a square or rectangle are equal. 1 - 1 - 0 - BH

(b) The diagonals of a square (but not those of a rectangle) intersect at right angles. 1 - 0 - 0 - B

(c) The diagonals of a square and of a rectangle bisect each other. 1 - 0 - 0 - B

(a)-(b)-(c)

(d) The angles of a square are bisected by the diagonals of the square. 1 - 0 - 0 - B

(8) Rhombus- The diagonals of a rhombus bisect each other at right angles, and the diagonals bisect the angles. 2 - 0 - 0 - FB

(9) Isosceles trapezium - In an isosceles trapezium: 1- 0 - 0 - B

(a) non-parallel sides are equal,

(b) base angles are equal,

(c) diagonals are equal.

XXXII Rounding Off of Numbers

(1) Numbers may be rounded off to any desired number of significant figures. 0 - 2 - 0 - DA

(2) Numbers may be rounded off to estimate an answer before working a problem. 0 - 0 - 1 - J

XXXIII Ratio

(1) Two numbers may be compared by ratio. 0 - 3 - 2

(2) Ratio (an abstract number) may be expressed as:

(a) an indicated division, as 3 ÷ 5, 0 - 1 - 1 - BD

(b) a fraction, as 3/5, 0 - 1 - 1 - HD

(c) in ratio form, as 3 : 5, 0 - 1 - 1 - HD

(d) as a quotient, as 0.6, 0 - 0 - 1 - D

(e) as a per cent, as 60%, 0 - 0 - 1 - D

(3) An increase or decrease may be described as a rate per cent of the given number. 0 - 1 - 1 - BD

(4) In every proportion, the product of the extremes is equal to the product of the means. 0 - 0 - 1 - E

XXXIV Square Root	(a)-(b)-(c)
(1) The square root of a number is one of its two equal factors,	1 - 3 - 6
(a) found by factoring,	0 - 2 - 0 - HD
(b) found by inspection,	1 - 2 - 0 - BFD
(c) found from a table of square roots,	0 - 4 - 1 - FDBAJ
(d) found by computation (explanation of the method by an illustration).	0 - 4 - 1 - HBAJD
(2) In square root the index 2 is omitted as $\sqrt[2]{a}$ is written $\sqrt{a}$.	3 - 3 - 0
(3) Interpolation may be done in a table of square roots.	1 - 1 - 0
(4) $\sqrt{ab} = \sqrt{a}\sqrt{b}$.	0 - 0 - 1
(5) The cube root of a number is one of its three equal factors, or generally $\sqrt[n]{x^n} = x$.	0 - 1 - 0

XXXV Signed Numbers	
(1) Signed numbers represent magnitudes.	1 - 8 - 0
(2) Signed numbers are directed numbers, negative numbers being opposite in direction to positive numbers.	1 - 8 - 0
(3) Signed numbers may be represented on a number scale.	1 - 8 - 0

XXXVI Scale Drawing	
(1) The actual dimensions of an object may be represented in a drawing by the use of a convenient scale.	0 - 4 - 0
(2) Scale drawing work makes use of the properties of similar figures.	1 - 2 - 0

XXXVII Significant Figures - Rule for: (a)-(b)-(c)

Keep in the product the same number of figures as in the factor having the smallest number of figures if the product of the digits on the left of each factor is more than 10, or if the product of the two digits on the left of each factor is more than 1000. Otherwise keep one figure less in the product. 1 - 0 - 1

XXXVIII Straight Line

(1) Of all straight lines drawn from a given point to a given straight line, the perpendicular is the least. 0 - 0 - 3 - HGK

(2) If a straight line is the shortest distance from a point to a line, then it is perpendicular to the line. 0 - 0 - 3 - HGK

(3) Only one perpendicular can be drawn to a straight line from a point outside that line. 0 - 0 - 3 - CGK

(4) Two obliques from a point of a perpendicular which cut the line at equal distances from the foot of a perpendicular, are equal. 0 - 0 - 2 - GK

(5) If one oblique cuts the line at a greater distance from the foot of the perpendicular than the other, it is greater in length. 0 - 0 - 2 - GK

(6) A straight line is determined by two points. 0 - 1 - 0 - A

XXXIX Similar Figures

(1) When two figures are similar, their corresponding angles are equal. 1 - 1 - 2 - ADFJ

(2) When two figures are similar, the ratio of any two corresponding sides equals the ratio of any other pair of corresponding sides. 1 - 1 - 2 - ADFJ

(3) Rectilineal figures are similar when

(a) they are equiangular. 0 - 1 - 3 - AGE

(b) sides about equal angles are proportional. 0 - 1 - 3 - AGE

(4) If for a given figure there is an axis of symmetry, then for every point P in the figure there is a corresponding point Q such that PQ is perpendicular to the axis and is bisected by it. 1 - 2 - 0 - AFD

(5) If for a given figure there is a centre of symmetry, then for every point P in the figure there is a corresponding point Q such that PQ is bisected at the centre of symmetry. 0 - 2 - 0 - FD

(6) The areas of similar figures are proportional to the squares of corresponding lines of the surfaces. 0 - 0 - 2 - GE

XL <u>Trigonometric Functions</u>

(1) As the angle increases in magnitude,

(a) the tangent of the angle increases, 0- 1 - 1 - DJ

(b) the sine of the angle increases, 0 -2 - 0 - DJ

(c) the cosine of the angle decreases. 0 - 2 - 0 -DJ

(2) The size of the trigonometric function depends on the size of the angle. 0 - 1 - 1 -DJ

(3) For any one given angle the tangent, the sine or the cosine always have constant values. 0 - 0 - 1 -J

(4) Trigonometric functions may be found from a table. 0 - 1 -1 - DJ

(5) $\tan 45^\circ = 1$ $\tan 60^\circ = \sqrt{3}$ $\tan 30^\circ = 1/\sqrt{3}$ 0- 0 - 1 -J
$\sin 45^\circ = 1/\sqrt{2}$ $\sin 60^\circ = \sqrt{3}/2$ $\sin 30^\circ = \frac{1}{2}$
$\cos 45^\circ = 1/\sqrt{2}$ $\cos 60^\circ = 1/2$ $\cos 30^\circ = \sqrt{3}/2$

XLI Triangles	(a)-(b)-(c)
(1) An equilateral triangle is equiangular.	0 - 2 - 2 - HDGK
(2) If a triangle is equiangular it is equilateral.	0 - 0 - 2 - GK
(3) If one side of a triangle is greater than another, then the angle opposite the greater side is greater than the angle opposite the less.	0 - 0 - 3 - HGK
(4) If one angle of a triangle is greater than another, then the side opposite to the greater angle is greater than the side opposite to the less.	0 - 0 - 3 - HGK
(5) The sum of two sides of a triangle is greater than the third side.	0 - 0 - 2 - GK
(6) The shortest distance between two points is the straight line which joins them.	0 - 0 - 2 - GK
(7) The acute angles of a right triangle are complementary.	0 - 0 - 2 - GK
(8) The perpendicular bisectors of the sides of a triangle are concurrent.	0 - 0 - 1 - K
(9) The bisectors of the angles of a triangle are concurrent.	0 - 0 - 1 - K
(10) The medians of a triangle are concurrent.	0 - 0 - 1 - K
(11) The medians of a triangle cut each other at a point of trisection, the greater segment in each being towards the angular point.	0 - 0 - 1 - K

XLII Congruence of Triangles	
(1) If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, the triangles are congruent.	0 - 2 - 5

(a)-(b)-(c)

(2) If two triangles have two angles and a side of one equal respectively to two angles and the corresponding side of the other, the triangles are congruent. 0 - 2 - 4

(3) If two triangles have three sides of one equal respectively to three sides of the other, the triangles are congruent. 0 - 1 - 5

(4) Congruent triangles have corresponding sides and corresponding angles equal. 0- 1 - 1

(5) If two right-angled triangles have the hypotenuse and one side of one equal to the hypotenuse and the corresponding side of the other, the triangles are congruent. 0 - 0- 3

(6) If two triangles have two sides of one equal to two sides of the other, each to each, but the angle included by the two sides of one greater than the angle included by the corresponding sides of the other, then the base of that which has the greater angle is greater than the base of the other. 0- 0 - 1

(7) If two triangles have two sides of one equal to two sides of the other, each to each, but the base of one greater than the base of the other, then the angle contained by the sides of that which has the greater base is greater than the angle contained by the corresponding sides of the other. 0 - 0 - 1

XLIII Isosceles Triangles

(1) The angles at the base of an isosceles triangle are equal. 0 - 3 - 5

(2) If the angles at the base of a triangle are equal, the sides opposite to those angles are equal. 0 - 1 - 4

(3) If the equal sides of an isosceles triangle are produced, the exterior angles are equal. 0 - 0 - 2 - GK

(4) An isosceles triangle is symmetrical about the bisector of the vertical angle. 0- 0 - 2 - GK

(a)-(b)-(c)

(5) An equilateral triangle is symmetrical about the bisector of any one of its angles. 0 - 0 - 2 - GK

XLIV <u>Angles of a Triangle, Quadrilateral</u>

(1) The sum of the angles of a triangle is equal to two right angles. 0 - 2 - 5

(2) If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two opposite interior angles. 0 - 0 - 5

(3) If A,B,C, denote the number of degrees in the angles of a triangle, then $A + B + C = 180^o$. 0 - 1 - 3

(4) If one side of triangle is produced the exterior angle so formed is greater than either of the interior and opposite angles. 0 - 0 - 3 - CGK

(5) If two angles of one triangle are respectively equal to two angles of another triangle, the third angles are equal. 0 - 0 - 3 - CGK

(6) If one angle of a triangle is equal to the sum of the other two angles, that angle is a right angle. 0 - 0 - 3 - CGK

(7) The sum of the exterior angles of a triangle is 360^o. 1 - 0 - 1 - CH

(8) Any two angles of triangle are together less than two right angles. 0 - 0 - 2 - GK

(9) Every triangle has two acute angles. 0 - 0 - 2 - GK

(10) The sum of the interior angles of a quadrilateral is 360^o. 0 - 2 - 2 - HCGK

(11) The sum of the interior angles of a polygon of N sides is (2N - 4) right angles. 0 - 1 - 2 - FGK

(12) The sum of the exterior angles of a polygon is 8 right angles. 0 - 1 - 2 - FGK

(13) The sum of the exterior angles of a quadrilateral is 360^o. 0 - 0 - 1 - H

XLV <u>Socialized Mathematics</u> (a)-(b)-(c)

This section gives some of the "socialized" information or generalizations offered in some of the more "socialized" texts.

(1) Thrift and Investment-

(a) Thrifty people keep account of their receipts and expenditures.	0 - 0 - 1 - J
(b) Thrifty people save a definite amount of money each month. They keep bank accounts.	0 - 1 - 1 - DJ
(c) Thrifty people do not make hasty investments. High rates of interest indicate a large element of risk.	0 - 1 - 1 - DJ
(d) Money saved in small amounts is put in a bank, in larger amounts is invested.	0 - 1 - 1 - DH

(2) Sales and Discounts-

(a) Sometimes money is saved by taking advantage of discount sales: (i) by paying cash to receive cash discounts. (ii) by paying bills to receive discounts.	0 - 0 - 1 - J
(b) A discount is usually stated as a per cent of the list price.	0 - 3 - 0 - CGK
(c) Successive discounts may be deducted in any order.	0 - 3 - 0 - CGK
(d) Trade discount is used to meet competition, induce buying in quantity, meet price changes in material or labor, sell out dead stock, make new catalogues unnecessary, when prices change.	0 - 0 - 1 - H

(3) Instalment Buying-

(a) Things bought on the instalment plan cost more than when paid for in cash.	0 - 1 - 0 - J
(b) People often buy things on instalment they do not need nor can afford.	0 - 1 - 0 - J

(c) The rate of interest on instalment buying can be calculated. The rate is usually high. 0 - 1 - 0 - J

(4) Investing in stocks and bonds-

(a) Investments not fully considered from every angle usually mean loss of money. Speculation is dangerous. 0 - 0 - 1 - J

(b) When a corporation is operating successfully, is paying high dividends, has good prospects for the future, the price of the shares tends to go up. 0 - 0 - 1 - J

(c) The price of the stocks varies with the value of money. 0- 1 - 0 - G

(d) Small dividends and poor prospects for the future tend to drive the price of shares down. 0 - 0 - 1 - J

(e) Safety of the money invested and permanence of reasonable dividends are the important things to consider before buying stocks. A corporation can pay dividends only if it makes profits. 0 - 0 - 1 - J

(5) Shares-

(a) When shares are bought, the brokerage is added to the cost of the shares to find the total cost. 0 - 1 - 1 - GJ

(b) When shares are sold, the brokerage is subtracted from the selling price to find the net amount. 0 - 1 - 1 - GJ

(c) The dividend on a share is expressed as a per cent of the par value of the share. 0 - 0 - 1 - J

(d) Stocks are named by the per cent paid. Bank stocks have double liability. 0 - 1 - 0 - G

(e) The rate of income on a share is the per cent the dividend is of the market price of the share. 0 - 0 - 1 - J

(6) Bonds- (a)-(b)-(c)

(a) The interest on a bond is a fixed per cent of the par value, or face value. This is an advantage over stocks. This interest is always the same, whether the market price is above or below par. When a bond sells below par, the rate of income is greater than the rate specified on the bond. When a bond sells above par, the rate of income is less than the rate specified on the bond. 0 - 2 - 1 - BGJ

(b) All bonds are not equally attractive as investments, as the greater the security, the safer the bond. 0- 0- 1 - J

(c) The total cost of the bond is made up of three items: the market price, the brokerage, the accrued interest. 0 - 0 - 1 - J

(d) Bonds are a safer form of investment than stocks, although not all bonds are equally secure. 0- 0- 1 - H

(7) Stocks and bonds compared-

(a) In stocks, the owner is part owner of the corporation, generally has a vote in the election of directors, dividends are not due until declared by directors, dividends depend on the earning power of the corporation, Stocks are frequently purchased for speculation. 0 - 0 - 1 - G

(b) In bonds, the owner is a creditor of the corporation, generally has not a vote, dividends (interest) are payable regularly, the interest is fixed. Bonds are generally purchased for investment. 0 - 0 - 1 - G

(8) Profit and loss - The rate per cent of profit may be calculated on cost price or on selling price. 0- 0- 1 - H

(9) Commission is always calculated on on the amount of money employed in the transaction. 0 - 3 - 0 - EGK

(10) Insurance-	(a)-(b)-(c)
(a) The rate of life insurance depends on the age of the insured person.	0 - 0 - 1 - H
(b) Life insurance cost is determined by mortality cost or death claims, reserve for cash surrender value or face of policies, expense of management.	0 - 0 - 3 - EGK
(c) Fire insurance premium rates are determined by the kind of fire protection, kind of building, proximity to and construction of adjoining buildings.	0 - 1 - 3 - DEGK
(11) Taxation-	
(a) Municipalities raise money by a direct tax on the land.	0 - 2 - 1 - DBH
(b) The Dominion Government raises money by customs and duties (these are used also to increase the price of cheap foreign goods to protect the home market).	0 - 1 - 1 - DH
(c) The Dominion Government raises money by the income tax.	0 - 2 - 1 - DBH
(d) The tax rate is determined from the total assessed valuation and the amount of the tax to be raised.	0 - 2 - 3 - HBEGK
(e) Taxes may be expressed as:	
(i) rate per cent of the assessed valuation,	0 - 1 - 1 - HB
(ii) dollars on \$100 of assessed valuation,	0 - 0 - 1 - B
(iii) dollars on \$1000 of assessed valuation,	0 - 0 - 1 - B
(iv) mills on \$1 of assessed valuation.	0 - 1 - 1 - HB
(12) Banks provide a safe place to deposit money, to pay bills by cheques, to cash cheques on other banks, pay interest on savings accounts, issue traveller's cheques, and letters of credit, issue bank notes.	0 - 0 - 2 - HE

(13) Exchange-	(a)-(b)-(c)
(a) A merchant may pay a debt by personal cheque, bank draft, postal money order, express money order, telegraphic money order, commercial draft.	0 - 0 - 1 - G
(b) The Pound equals \$4 4/9 (old par) or \$4.86 2/3 (new par).	0 - 0 - 1 - G
(c) The Course of Exchange is determined by the balance of trade between two countries.	0 - 1 - 0 - G
(14) Interest-	
(a) Compound interest may be found from a table.	0 - 1 - 4 - DHBJK
(b) Interest compounded semiannually (or quarterly) at the same rate is equal to the interest compounded annually using one half (or one fourth) the rate for twice (or four times) as many periods as there are years.	0 - 1 - 2 - KDB
(c) Simple interest is less than compound interest.	0 - 2 - 0 - DJ
(d) Simple interest may be found from tables (using the denominator of the fraction always as 365, with the numerator the number of days in the part of the year).	0 - 1 - 2 - DHE

SUMMARY AND CONCLUSIONS.

In the preceding pages of this report, 455 generalizations under the 45 headings have been tabulated as those basic to the Grade Nine Mathematics curricula of the Intermediate and High Schools of Canada. Of the total frequencies, 8.9% were tabulated as (a) assumed to be true, and used without explanation or statement, 36.4% as (b) assumed to be true, but explained or stated informally, and 54.7% as (c) offered with a proof or stated formally often with an explanation.

TABLE SUMMARIZING FREQUENCY OF GENERALIZATIONS.

	Number of generalizations found in	Number of texts
	9	10
	32	9
	25	8
	21	7
	21	6
	30	5
	36	4
	64	3
	101	2
	116	1
Total	455	

That is, 9 generalizations were found in all 10 texts, 32 were found in only 9 of the 10, 25 in 8 and down through the table to 116 generalizations found in only one of the ten texts.

These 455 generalizations, including the socialized mathematics concepts are presented as the subject matter of the present Grade Nine curricula of Canada. In this report these generalizations have been distinguished from the 550 language terms given in Appendix III.

In tabulating the generalizations, an attempt was made by a letter designation to indicate the texts which gave some of the least common generalizations. This was done for the purpose of giving the reader an opportunity of noting that these groups of generalizations came from the same or different texts. Appendix I, which lists the texts by provinces, gives further comments on the nature of each text.

No attempt has been made in this report to indicate a logical or a psychological order in which these generalizations could be presented in the classroom;.nor has any attempt been made to suggest that the more common generalizations constitute "an ideal course" or the subject matter from which an ideal course could be compiled in mathematics for pupils at the Grade nine level.

APPENDIX I.

THE TEXTS.

The following ten texts or groups of texts were used in this study. This list was compiled from the 1943-44 Programmes of Studies for each of the nine provinces. A comment is given for each text indicating parts to be omitted and the general nature of the text.

British Columbia -

Dent's Junior Mathematics Book III for Grade IX by Gray and Reid. 1940 Dent's.

The entire text to be studied with Chapters XII, XIII, XIV, the part on Business Arithmetic optional. Both a semi-logical and a socialized treatment of subject matter.

Alberta-

Mathematics for Today by LaZerte and Betz. Ginn & Co.

The entire text to be studied. A logical treatment.

Saskatchewan -

Everyday Mathematics Grade Nine by Anderson and Shanklin. 1943 W.J.Gage.

The entire text to be studied with stress on the topics listed in the Programme of Studies. Mainly a socialized, with some semi-logical treatment of subject matter.

Manitoba -

Mathematics in Action Book III by Hart, Jahn and Robinson. 1943 Copp Clark Co.

The text used to study the units listed in the Course of Studies. A socialized treatment.

Ontario -

General Mathematics Book I by Jackson, Dean, Crawford. 1942 MacMillan.

The entire text to be studied. A formal treatment.

Quebec, Protestant -

1. Geometry for High Schools by Lougheed & Workman.
   1935 MacMillan.

   Pages 1 - 91 to be studied. A logical treatment.

2. Progressive High School Algebra by Hart.
   1940 Copp Clark.

   Pages 157-184, 194-203, 210-263 to be studied.
   A logical treatment.

3. Arithmetic - no text given. A review to be
   made of Grade VIII Arithmetic.

Quebec, Catholic -

1. High School Algebra by Crawford
   1942 MacMillan.

   Pages 1 - 114 to be studied. A logical treatment.

2. Dominion High School Arithmetic
   1941 W.J.Gage.

   Pages 1 - 251 with Chapters 5,6,10,13,14 and
   pages 132-138, 218 omitted. Socialized treatment.

Nova Scotia -

Mathematics Grade IX, Highroads of Mathematics Book IV
by McCarthy and Highet. T.Nelson Sons.

Entire text to be studied. A socialized and
semi-logical treatment.

New Brunswick -

1. A School Geometry by Hall & Stevens
   1938 MacMillan

   Part I to be studied, omitting pages 65,95 and 98.
   A formal, logical treatment.

2. High School Algebra by Crawford 1942 MacMillan

   Chapters 1 - 12 to be studied. A logical treatment.

3. Dominion High School Arithmetic 1941 W.J.Gage.

   A study of per cent and its application to stocks
   and bonds, exchange, mensuration of surfaces, solids,
   metric system. A socialized treatment.

Prince Edward Island -

1. A School Geometry by Hall & Stevens 1938 MacMillan.

   Part I to be studied omitting Locus, Theorems 19,22. A formal, logical treatment.

2. High School Algebra by Crawford 1942 MacMillan

   Chapters 1 - 15 to be studied, omitting Chapters 7, 12, 13, 14. A logical treatment.

3. Dominion High School Arithmetic 1941 W.J.Gage

   A study of topics listed in the Course of Studies (see Appendix II)

APPENDIX II.

PROGRAMMES OF STUDIES.

The following notes were made from the Programmes of Studies for each of the nine provinces:

British Columbia - A complete discussion of the topics to be studied, listing objectives, teaching procedures, vocabulary to be stressed for each unit.

Alberta - Lists the text and gives a brief discussion of the treatment of Geometry at this stage.

Saskatchewan - Gives a brief statement of the objectives, indicating the stress on socialized mathematics. The topics to be studied are ; geometrical construction and inductive reasoning, formula, equation, ratio, indirect measurement, application of mathematics to simple mensuration and business transactions, signed numbers, products, simple factoring, simple graphs, practical farm arithmetic.

Manitoba - An outline of the units to be studied and the number of periods to be spent on each unit.

Ontario - Lists the objectives and the texts.

Quebec, Protestant - Lists the texts.

Quebec, Catholic - Lists the texts.

Nova Scotia - Lists the texts.

New Brunswick - Lists the texts.

Prince Edward Island - Lists the texts and the topics to be studied in the Arithmetic, which are:

1. Speed and accuracy in fundamentals - integral numbers, fractions, decimals, per cent.

2. Review of Grade VIII - denominate numbers, fractions, decimals, mensuration, per cent.

3. Compound interest - use of tables and some use of the long method.

4. Use of graphs.

5. Equations - using letters for unknowns, simple equations in one unknown.

6. Positive and negative numbers, fundamental operations.

7. Mensuration - area of triangle, parallelograms, curved surface of cylinder.

APPENDIX III.

LANGUAGE.

The following terms[1] were selected arbitrarily as language and not generalizations. The list does not necessarily include all mathematical terms used in the texts. The terms are listed by topics, alphabetically arranged,

absolute value

addition - addend
- indicated addition

algebra - algebraic

altitude

Approximation

area

axiom

angle - vertex
arm
degree
minute
second
included, contained
supplementary,
supplement
complementary
complement
elevation
depression
straight
right
acute
obtuse
reflex
exterior
adjacent
vertically opposite
corresponding
(in triangles)

bisector - right
internal (angle)
external (angle)

brick work - superficial foot

cash

circle - semicircle
circumference
arc
chord
sector
diameter
radius
tangent
point of contact
concentric
centre
inscribe
circumscribe
segment
quadrant

coefficient - numerical
literal

collect (terms)

combine (terms)

corollary

cube root

cyclic order (of denominators)

---

1. 547 or approximately 550 language terms are listed in Appendix III

decimal - decimal point
recurring
mixed recurring

density

division - divisor
- trial
true
dividend
quotient
remainder

discount

digit

due

definition

diagonal

equation - unknown
root
linear
simultaneous
sdution
solve
satisfy
verification
left side
right side
identity
quadratic
consistent
inconsistent
dependent
independent
determinate
indeterminate

evaluation - evaluate

expression - terms
simple
compound
monomial
binomial
trinomial
polynomial
multinomial

factor - literal
numerical
prime
highest common

figure - plane figure

formula - subject of

function

fraction - common
vulgar
numerator
denominator
-common, lowest common
terms
proper
improper
compound
complex
lowest terms
equivalent
aliquot parts
cancellation

gallon - Imperial
American

geometry - geometric

graph - bar
circle
pictograph
rectangular distribution
broken line
curved line
coordinate
axis
ordinate
abscissa
origin

hundredths

inspection

length

linear

line - horizontal
 vertical
 oblique
 straight
 broken
 curved
 concurrent lines
 segment, finite

locus

longitude

measurement - unit of
 English ton
 fathom
 link
 chain
 acre
 watt
 kilowatt-hour
 board foot

metric system - millimetre
 centimetre
 decimetre
 metre
 dekametre
 hektometre
 litre
 stere
 gram
 kilogram
 metric ton
 are
 myria-
 mega-
 micro-

mile - nautical mile

multiplication - multiplier
 multiplicand

multiple - least common
 lowest common

negative

nomenclature

notation

numeration

number - arithmetic
 algebraic
 literal
 signed
 directed
 negative
 positive
 mixed
 whole
 prime
 round
 integer

orthogonal projection

order - ascending
 descending

parentheses - brackets

parallel lines - cutting line
 transversal
 alternate angles
 corresponding angles
 exterior angles
 interior angles
 interior and opposite angles

perpendicular

per cent

perimeter

plane

point

positive

postulate

possible error - probable error

product

proportion - proportional
terms
extremes
means
simple

prime - number
factor

power - base
exponent
index, indices

pythagorean

proposition - theorem
problem
enunciation
-general
particular
hypothesis
conclusion
construction
proof
-indirect
method of
converse

quantity - known
unknown

ratio - antecedent
consequent

reciprocal

rectilinear figures -
-triangle
quadrilateral
trapezium
trapezoid
parallelogram
rectangle
square
rhombus
polygon
-penta
hexa
octa
deca
equilateral
equiangular
regular

roof - rise
run
pitch
square

scale

scale drawing

signs - operation
quality

significant figures

simplify

subtraction - subtract
subtrahend
minuend
indicated

surface - plane
curved

solid - cube
rectangular
rectangular parallelopiped
cuboid
cylinder - right circular
- lateral area of
cone - right circular
prism
sphere
pyramid - regular
triangular
square
pentagonal
hexagonal

specific gravity

symbol

symmetry - axis of
point of

square

superposition

stairs - rise
 riser
 nosing
 run

stone work - perch

square root - root
 root sign
 principal
 radical sign

terms - like
 unlike

temperature - Fahrenheit
 Centigrade
 degree

time - standard time

table of values

trigonometric ratios -
 -tangent
 sine
 cosine

triangle - sides
 base
 vertex
 vertices
 congruent
 equilateral
 equiangular
 isosceles
 scalene
 right-angled
 obtuse-angled
 acute-angled
 similar
 hypotenuse
 median
 corresponding angles

verify

variable

value - absolute

vinculum

Socialized Mathematics Terms

banking - deposit
 deposit slip
 bank book
 current account
 savings account
 cheque
 cheque book
 endorse
 exchange
 credit
 promissory note
 bank discount

bond - par value
 coupon
 rate
 register
 mature
 mortgage bond
 market price
 bond quotations
 accrued interest

customs - duties
 Customs House
 Ports of entry
 excise
 tariffs
 ad valorem
 specific

commission - principal
 consignor
 consignee
 shipment
 proceeds - gross
 net

exchange - domestic
 foreign
 bill of
 rate of
 par of
 draft
 balance of trade
 course of exchange
 arbitration of exchange
 simple
 compound

instalment buying

insurance - property
personal
premium
policy
face value
face of the policy
loan value
cash surrender value
ordinary life
limited payment
endowment
term

interest - rate
simple
compound
principal
amount
annual
semi-annual
quarterly

investment

discount - commercial
trade
successive

draft - drawer
drawee
payee
debtor
creditor
acceptor

notes - promissory
face
time
date of maturity
discount, terms of
proceeds
days of grace
maker
payee
negotiable
non-negotiable
endorsement

mortgage

price - list
catalogue
invoice

partnership - partners
capital
assets
liabilities

profit and loss - net cost
overhead
marked price
selling price
gain

purchases-

taxation - direct
indirect
budget
income tax
normal tax
surtax
additional tax
exemptions
dependents
appraise
mill rate
assessor
assessed
assessment
valuation
real estate tax
personal property tax
inheritance tax
poll tax

thrift-

sales

stocks - capital
share
shareholder
stock certificate
par value
nominal value
market value
capitalized
dividends
preferred
common
stock exchange
quotations
price change
brokerage
broker
corporation
joint stock company
charter
stockholder
discount
premium